

**TEST PAPER OF JEE(MAIN) EXAMINATION – 2019****(Held On SATURDAY 12<sup>th</sup> JANUARY., 2019) TIME : 02 : 30 PM To 05 : 30 PM****MATHEMATICS**

1. Let  $Z$  be the set of integers. If

$$A = \{x \in Z : 2(x+2)(x^2 - 5x + 6)\} = 1 \quad \text{and}$$

$B = \{x \in Z : -3 < 2x - 1 < 9\}$ , then the number of subsets of the set  $A \times B$ , is :

- (1)  $2^{18}$       (2)  $2^{10}$       (3)  $2^{15}$       (4)  $2^{12}$

**Ans (3)**

**Sol.**  $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$

$$2^{(x+2)(x^2-5x+6)} = 2^0 \Rightarrow x = -2, 2, 3$$

$$A = \{-2, 2, 3\}$$

$$B = \{x \in Z : -3 < 2x - 1 < 9\}$$

$$B = \{0, 1, 2, 3, 4\}$$

$A \times B$  has 15 elements so number of subsets of  $A \times B$  is  $2^{15}$ .

2. If  $\sin^4 \alpha + 4 \cos^4 \beta + 2 = 4\sqrt{2} \sin \alpha \cos \beta$  ;

$\alpha, \beta \in [0, \pi]$ , then  $\cos(\alpha + \beta) - \cos(\alpha - \beta)$  is equal to :

- (1) 0      (2)  $-\sqrt{2}$       (3) -1      (4)  $\sqrt{2}$

**Ans (2)**

**Sol.** A.M.  $\geq$  G.M.

$$\frac{\sin^4 \alpha + 4 \cos^4 \beta + 1 + 1}{4} \geq (\sin^4 \alpha \cdot 4 \cos^4 \beta \cdot 1 \cdot 1)^{\frac{1}{4}}$$

$$\sin^4 \alpha + 4 \cos^4 \beta + 2 \geq 4\sqrt{2} \sin \alpha \cos \beta$$

given that  $\sin^4 \alpha + 4 \cos^4 \beta + 2 = 4\sqrt{2} \sin \alpha \cos \beta$

$$\Rightarrow \text{A.M.} = \text{G.M.} \Rightarrow \sin^4 \alpha = 1 = 4 \cos^4 \beta$$

$$\sin \alpha = 1, \cos \beta = \pm \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin \beta = \frac{1}{\sqrt{2}} \text{ as } \beta \in [0, \pi]$$

$$\cos(\alpha + \beta) - \cos(\alpha - \beta) = -2 \sin \alpha \sin \beta = -\sqrt{2}$$



3. If an angle between the line,  $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$

and the plane,  $x - 2y - kz = 3$  is  $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ ,

then a value of  $k$  is:

- (1)  $-\frac{5}{3}$       (2)  $\sqrt{\frac{3}{5}}$       (3)  $\sqrt{\frac{5}{3}}$       (4)  $-\frac{3}{5}$

**Ans (3)**

**Sol.** DR's of line are 2, 1, -2

normal vector of plane is  $\hat{i} - 2\hat{j} - k\hat{k}$

$$\sin \alpha = \frac{(2\hat{i} + \hat{j} - 2\hat{k}) \cdot (\hat{i} - 2\hat{j} - k\hat{k})}{3\sqrt{1+4+k^2}}$$

$$\sin \alpha = \frac{2k}{3\sqrt{k^2+5}} \quad \dots\dots(1)$$

$$\cos \alpha = \frac{2\sqrt{2}}{3} \quad \dots\dots(2)$$

$$(1)^2 + (2)^2 = 1 \Rightarrow k^2 = \frac{5}{3}$$

4. If a straight line passing through the point  $P(-3, 4)$  is such that its intercepted portion between the coordinate axes is bisected at  $P$ , then its equation is :

(1)  $x - y + 7 = 0$

(2)  $3x - 4y + 25 = 0$

(3)  $4x + 3y = 0$

(4)  $4x - 3y + 24 = 0$

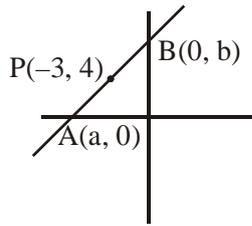
**Ans (4)**

Click Here for JEE Mains & Advanced Free Online Test Practice

<https://examination-system.shineitsolutions.in/jee-mains-and-advanced-examination-system/student-login.php>



Sol.



Let the line be  $\frac{x}{a} + \frac{y}{b} = 1$

$$(-3, 4) = \left(\frac{a}{2}, \frac{b}{2}\right)$$

$$a = -6, b = 8$$

equation of line is  $4x - 3y + 24 = 0$

5. The integral  $\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx$  is equal to :

(where C is a constant of integration)

$$(1) \frac{x^4}{(2x^4 + 3x^2 + 1)^3} + C$$

$$(2) \frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C$$

$$(3) \frac{x^4}{6(2x^4 + 3x^2 + 1)^3} + C$$

$$(4) \frac{x^{12}}{(2x^4 + 3x^2 + 1)^3} + C$$

Ans (2)

Sol.  $\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx$

$$\int \frac{\left(\frac{3}{x^3} + \frac{2}{x^5}\right) dx}{\left(2 + \frac{3}{x^2} + \frac{1}{x^4}\right)^4}$$

$$\text{Let } \left(2 + \frac{3}{x^2} + \frac{1}{x^4}\right) = t$$

$$-\frac{1}{2} \int \frac{dt}{t^4} = \frac{1}{6t^3} + C \Rightarrow \frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C$$

6. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is :

- (1) 9      (2) 11      (3) 12      (4) 7

Ans (3)

Sol. Let m-men, 2-women

$${}^m C_2 \times 2 = {}^m C_1 \cdot {}^2 C_1 \cdot 2 + 84$$

$$m^2 - 5m - 84 = 0 \Rightarrow (m - 12)(m + 7) = 0$$

$$m = 12$$

7. If the function  $f$  given by  $f(x) = x^3 - 3(a - 2)x^2 + 3ax + 7$ , for some  $a \in \mathbb{R}$  is increasing in  $(0, 1]$  and decreasing in  $[1, 5)$ , then a root of the equation,

$$\frac{f(x) - 14}{(x - 1)^2} = 0 (x \neq 1) \text{ is :}$$

- (1) 6      (2) 5      (3) 7      (4) -7

Ans (3)

Sol.  $f(x) = 3x^2 - 6(a - 2)x + 3a$

$$f'(x) \geq 0 \quad \forall x \in (0, 1]$$

$$f'(x) \leq 0 \quad \forall x \in [1, 5)$$

$$\Rightarrow f'(x) = 0 \text{ at } x = 1 \Rightarrow a = 5$$

$$f(x) - 14 = (x - 1)^2 (x - 7)$$

$$\frac{f(x) - 14}{(x - 1)^2} = x - 7$$

8. Let  $f$  be a differentiable function such that  $f(1) = 2$  and  $f'(x) = f(x)$  for all  $x \in \mathbb{R}$ . If  $h(x) = f(f(x))$ , then  $h'(1)$  is equal to :

- (1)  $4e$       (2)  $4e^2$       (3)  $2e$       (4)  $2e^2$

Ans (1)

Sol.  $\frac{f'(x)}{f(x)} = 1 \quad \forall x \in \mathbb{R}$

Integrate & use  $f(1) = 2$

$$f(x) = 2e^{x-1} \Rightarrow f'(x) = 2e^{x-1}$$

$$h(x) = f(f(x)) \Rightarrow h'(x) = f'(f(x)) f'(x)$$

$$h'(1) = f'(f(1)) f'(1)$$

$$= f'(2) f'(1)$$

$$= 2e \cdot 2 = 4e$$



9. The tangent to the curve  $y = x^2 - 5x + 5$ , parallel to the line  $2y = 4x + 1$ , also passes through the point.

(1)  $\left(\frac{1}{4}, \frac{7}{2}\right)$                       (2)  $\left(\frac{7}{2}, \frac{1}{4}\right)$

(3)  $\left(-\frac{1}{8}, 7\right)$                       (4)  $\left(\frac{1}{8}, -7\right)$

Ans (4)

Sol.  $y = x^2 - 5x + 5$

$$\frac{dy}{dx} = 2x - 5 = 2 \Rightarrow x = \frac{7}{2}$$

$$\text{at } x = \frac{7}{2}, y = \frac{-1}{4}$$

Equation of tangent at  $\left(\frac{7}{2}, \frac{-1}{4}\right)$  is  $2x - y - \frac{29}{4} = 0$

Now check options

$$x = \frac{1}{8}, y = -7$$

10. Let S be the set of all real values of  $\lambda$  such that a plane passing through the points  $(-\lambda^2, 1, 1)$ ,  $(1, -\lambda^2, 1)$  and  $(1, 1, -\lambda^2)$  also passes through the point  $(-1, -1, 1)$ . Then S is equal to :

(1)  $\{\sqrt{3}\}$                       (2)  $\{\sqrt{3}, -\sqrt{3}\}$

(3)  $\{1, -1\}$                       (4)  $\{3, -3\}$

Ans (2)

Sol. All four points are coplaner so

$$\begin{vmatrix} 1-\lambda^2 & 2 & 0 \\ 2 & -\lambda^2+1 & 0 \\ 2 & 2 & -\lambda^2-1 \end{vmatrix} = 0$$

$$(\lambda^2 + 1)^2 (3 - \lambda^2) = 0$$

$$\lambda = \pm\sqrt{3}$$



11. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is :

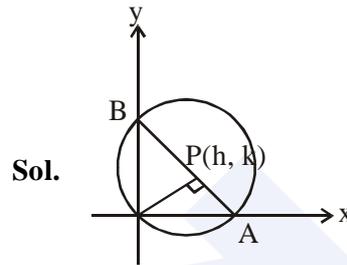
(1)  $(x^2 + y^2)^2 = 4R^2x^2y^2$

(2)  $(x^2 + y^2)(x + y) = R^2xy$

(3)  $(x^2 + y^2)^3 = 4R^2x^2y^2$

(4)  $(x^2 + y^2)^2 = 4R^2x^2y^2$

Ans (3)



Sol.

$$\text{Slope of AB} = \frac{-h}{k}$$

Equation of AB is  $hx + ky = h^2 + k^2$

$$A\left(\frac{h^2 + k^2}{h}, 0\right), B\left(0, \frac{h^2 + k^2}{k}\right)$$

$$AB = 2R$$

$$\Rightarrow (h^2 + k^2)^3 = 4R^2h^2k^2$$

$$\Rightarrow (x^2 + y^2)^3 = 4R^2x^2y^2$$

12. The equation of a tangent to the parabola,  $x^2 = 8y$ , which makes an angle  $\theta$  with the positive direction of x-axis, is :

(1)  $x = y \cot \theta + 2 \tan \theta$                       (2)  $x = y \cot \theta - 2 \tan \theta$

(3)  $y = x \tan \theta - 2 \cot \theta$                       (4)  $y = x \tan \theta + 2 \cot \theta$

Ans (1)

Sol.  $x^2 = 8y$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{4} = \tan \theta$$

$$\therefore x_1 = 4 \tan \theta$$

$$y_1 = 2 \tan^2 \theta$$

Equation of tangent :-

$$y - 2 \tan^2 \theta = \tan \theta (x - 4 \tan \theta)$$

$$\Rightarrow x = y \cot \theta + 2 \tan \theta$$

Click Here for CBSE NEET & AIIMS  
Free Online Test Practice  
<https://examination-system.shineitsolutions.in/cbse-neet-and-aiims-examination-system/student-login.php>

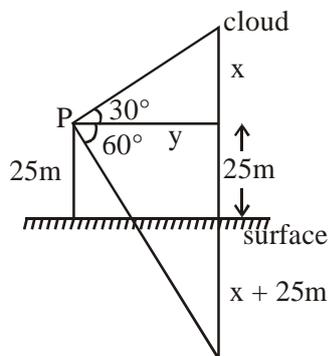


## JEE (Main) Examination–2019/Evening Session/12-01-2019

13. If the angle of elevation of a cloud from a point P which is 25 m above a lake be  $30^\circ$  and the angle of depression of reflection of the cloud in the lake from P be  $60^\circ$ , then the height of the cloud (in meters) from the surface of the lake is :

(1) 42      (2) 50      (3) 45      (4) 60

Ans (2)



Sol.

$$\tan 30^\circ = \frac{x}{y} \Rightarrow y = \sqrt{3}x \quad \dots(i)$$

$$\tan 60^\circ = \frac{25+x+25}{y}$$

$$\Rightarrow \sqrt{3}y = 50 + x$$

$$\Rightarrow 3x = 50 + x$$

$$\Rightarrow x = 25 \text{ m}$$

$\therefore$  Height of cloud from surface =  $25 + 25 = 50\text{m}$

14. The integral  $\int_1^e \left\{ \left( \frac{x}{e} \right)^{2x} - \left( \frac{e}{x} \right)^x \right\} \log_e x dx$  is equal to :

(1)  $\frac{1}{2} - e - \frac{1}{e^2}$       (2)  $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$

(3)  $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$       (4)  $\frac{3}{2} - e - \frac{1}{2e^2}$

Ans. (4)

$$\int_1^e \left( \frac{x}{e} \right)^{2x} \log_e x dx - \int_1^e \left( \frac{e}{x} \right)^x \log_e x dx$$

$$\text{Let } \left( \frac{x}{e} \right)^{2x} = t, \left( \frac{e}{x} \right)^x = v$$

$$= \frac{1}{2} \int_{\left(\frac{1}{e}\right)^2}^1 dt + \int_e^1 dv$$

$$= \frac{1}{2} \left( 1 - \frac{1}{e^2} \right) + (1 - e) = \frac{3}{2} - \frac{1}{2e^2} - e$$

$$15. \lim_{n \rightarrow \infty} \left( \frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \dots + \frac{1}{5n} \right)$$

is equal to :

(1)  $\frac{\pi}{4}$       (2)  $\tan^{-1}(2)$

(3)  $\tan^{-1}(3)$       (4)  $\frac{\pi}{2}$

Ans. (2)

$$\lim_{x \rightarrow \infty} \sum_{r=1}^{2n} \frac{n}{n^2+r^2}$$

$$\lim_{x \rightarrow \infty} \sum_{r=1}^{2n} \frac{1}{n \left( 1 + \frac{r^2}{n^2} \right)} = \int_0^2 \frac{dx}{1+x^2} = \tan^{-1} 2$$

16. The set of all values of  $\lambda$  for which the system of linear equations.

$$x - 2y - 2z = \lambda x$$

$$x + 2y + z = \lambda y$$

$$-x - y = \lambda z$$

has a non-trivial solution.

(1) contains more than two elements

(2) is a singleton

(3) is an empty set

(4) contains exactly two elements

Ans. (2)

$$\begin{vmatrix} \lambda-1 & 2 & 2 \\ 1 & 2-\lambda & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0 \Rightarrow (\lambda-1)^3 = 0 \Rightarrow \lambda = 1$$

17. If  ${}^nC_4$ ,  ${}^nC_5$  and  ${}^nC_6$  are in A.P., then n can be:

(1) 14      (2) 11      (3) 9      (4) 12

Ans. (1)

$$2 \cdot {}^nC_5 = {}^nC_4 + {}^nC_6$$

$$2 \cdot \frac{|n|}{|5|n-5} = \frac{|n|}{|4|n-4} + \frac{|n|}{|6|n-6}$$

$$\frac{2}{5} \cdot \frac{1}{n-5} = \frac{1}{(n-4)(n-5)} + \frac{1}{30}$$

n = 14 satisfying equation.



18. Let  $\vec{a}, \vec{b}$  and  $\vec{c}$  be three unit vectors, out of which vectors  $\vec{b}$  and  $\vec{c}$  are non-parallel. If  $\alpha$  and  $\beta$  are the angles which vector  $\vec{a}$  makes with vectors  $\vec{b}$  and  $\vec{c}$  respectively and  $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2} \vec{b}$ , then

$|\alpha - \beta|$  is equal to :

- (1)  $60^\circ$  (2)  $30^\circ$  (3)  $90^\circ$  (4)  $45^\circ$

Ans. (2)

$$(\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c} = \frac{1}{2} \vec{b}$$

$\therefore \vec{b}$  &  $\vec{c}$  are linearly independent

$$\therefore \vec{a} \cdot \vec{c} = \frac{1}{2} \quad \& \quad \vec{a} \cdot \vec{b} = 0$$

(All given vectors are unit vectors)

$$\therefore \vec{a} \wedge \vec{c} = 60^\circ \quad \& \quad \vec{a} \wedge \vec{b} = 90^\circ$$

$$\therefore |\alpha - \beta| = 30^\circ$$

19. If  $A = \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix}$ ; then for all

$\theta \in \left( \frac{3\pi}{4}, \frac{5\pi}{4} \right)$ ,  $\det(A)$  lies in the interval :

(1)  $\left[ \frac{5}{2}, 4 \right)$  (2)  $\left( \frac{3}{2}, 3 \right]$

(3)  $\left( 0, \frac{3}{2} \right]$  (4)  $\left( 1, \frac{5}{2} \right]$

Ans (2)

$$|A| = \begin{vmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{vmatrix}$$

$$= 2(1 + \sin^2 \theta)$$

$$\theta \in \left( \frac{3\pi}{4}, \frac{5\pi}{4} \right) \Rightarrow \frac{1}{\sqrt{2}} < \sin \theta < \frac{1}{\sqrt{2}}$$

$$\Rightarrow 0 \leq \sin^2 \theta < \frac{1}{2}$$

$$\therefore |A| \in [2, 3)$$

20.  $\lim_{x \rightarrow 1^-} \frac{\sqrt{\pi} - \sqrt{2 \sin^{-1} x}}{\sqrt{1-x}}$  is equal to :

- (1)  $\frac{1}{\sqrt{2\pi}}$  (2)  $\sqrt{\frac{\pi}{2}}$  (3)  $\sqrt{\frac{2}{\pi}}$  (4)  $\sqrt{\pi}$

Ans. (3)

$$\lim_{x \rightarrow 1^-} \frac{\sqrt{\pi} - \sqrt{2 \sin^{-1} x}}{\sqrt{1-x}} \times \frac{\sqrt{\pi} + \sqrt{2 \sin^{-1} x}}{\sqrt{\pi} + \sqrt{2 \sin^{-1} x}}$$

$$\lim_{x \rightarrow 1^-} \frac{2 \left( \frac{\pi}{2} - \sin^{-1} x \right)}{\sqrt{1-x} \cdot (\sqrt{\pi} + \sqrt{2 \sin^{-1} x})}$$

$$\lim_{x \rightarrow 1^-} \frac{2 \cos^{-1} x}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{\pi}}$$

Put  $x = \cos \theta$

$$\lim_{\theta \rightarrow 0^+} \frac{2\theta}{\sqrt{2} \sin \left( \frac{\theta}{2} \right)} \cdot \frac{1}{2\sqrt{\pi}} = \sqrt{\frac{2}{\pi}}$$

21. The expression  $\sim(\sim p \rightarrow q)$  is logically equivalent to :

- (1)  $\sim p \wedge \sim q$  (2)  $p \wedge q$   
(3)  $\sim p \wedge q$  (4)  $p \wedge \sim q$

Ans. (1)

p	q	$\sim p$	$\sim p \rightarrow q$	$\sim(\sim p \rightarrow q)$	$(\sim p \wedge \sim q)$
T	T	F	T	F	F
F	T	T	T	F	F
T	F	F	T	F	F
F	F	T	F	T	T

22. The total number of irrational terms in the binomial

expansion of  $(7^{1/5} - 3^{1/10})^{60}$  is :

- (1) 55 (2) 49 (3) 48 (4) 54

Ans. (4)

$$\text{General term } T_{r+1} = {}^{60}C_r \cdot 7^{\frac{60-r}{5}} \cdot 3^{\frac{r}{10}}$$

$\therefore$  for rational term,  $r = 0, 10, 20, 30, 40, 50, 60$

$\Rightarrow$  no of rational terms = 7

$\therefore$  number of irrational terms = 54



## JEE (Main) Examination–2019/Evening Session/12-01-2019



23. The mean and the variance of five observation are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then then absolute value of the difference of the other two observations, is :

- (1) 1      (2) 3      (3) 7      (4) 5

Ans. (3)

$$\text{mean } \bar{x} = 4, \sigma^2 = 5.2, n = 5, x_1 = 3, x_2 = 4 = x_3,$$

$$\sum x_i = 20$$

$$x_4 + x_5 = 9 \dots\dots(i)$$

$$\frac{\sum x_i^2}{x} - (\bar{x})^2 = \sigma^2 \Rightarrow \sum x_i^2 = 106$$

$$x_4^2 + x_5^2 = 65 \dots\dots(ii)$$

$$\text{Using (i) and (ii) } (x_4 - x_5)^2 = 49$$

$$|x_4 - x_5| = 7$$

24. If the sum of the first 15 terms of the

series  $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + \dots$  is

equal to 225 k, then k is equal to :

- (1) 9      (2) 27      (3) 108      (4) 54

Ans. (2)

$$S = \left(\frac{3}{4}\right)^3 + \left(\frac{6}{4}\right)^3 + \left(\frac{9}{4}\right)^3 + \left(\frac{12}{4}\right)^3 + \dots\dots\dots 15 \text{ term}$$

$$= \frac{27}{64} \sum_{r=1}^{15} r^3$$

$$= \frac{27}{64} \cdot \left[ \frac{15(15+1)}{2} \right]^2$$

$$= 225 K \text{ (Given in question)}$$

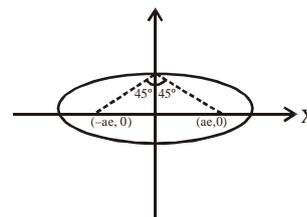
$$K = 27$$

25. Let S and S' be the foci of the ellipse and B be any one of the extremities of its minor axis. If  $\Delta S'BS$  is a right angled triangle with right angle at B and area  $(\Delta S'BS) = 8$  sq. units, then the length of a latus rectum of the ellipse is :

- (1)  $2\sqrt{2}$       (2) 2      (3) 4      (4)  $4\sqrt{2}$

Ans. (3)

$$m_{SB} \cdot m_{S'B} = -1$$



$$b^2 = a^2 e^2 \dots (i)$$

$$\frac{1}{2} S'B \cdot SB = 8$$

$$S'B \cdot SB = 16$$

$$a^2 e^2 + b^2 = 16 \dots (ii)$$

$$b^2 = a^2 (1 - e^2) \dots (iii)$$

$$\text{using (i), (ii), (iii) } a = 4$$

$$b = 2\sqrt{2}$$

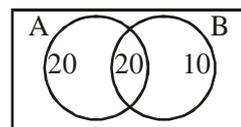
$$e = \frac{1}{\sqrt{2}}$$

$$\therefore \ell \text{ (L.R)} = \frac{2b^2}{a} = 4 \quad \boxed{\text{Ans.3}}$$

26. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither for NCC nor for NSS is :

- (1)  $\frac{2}{3}$       (2)  $\frac{1}{6}$       (3)  $\frac{1}{3}$       (4)  $\frac{5}{6}$

Ans. (2)



A  $\rightarrow$  opted NCC

B  $\rightarrow$  opted NSS

$$\therefore P \text{ (nither A nor B)} = \frac{10}{60} = \frac{1}{6}$$



27. The number of integral values of  $m$  for which the quadratic expression.

$(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$ ,  $x \in \mathbb{R}$ , is always positive, is :

- (1) 8            (2) 7            (3) 6            (4) 3

Ans. (2)

Expression is always positive if

$$2m + 1 > 0 \Rightarrow m > -\frac{1}{2}$$

&

$$D < 0 \Rightarrow m^2 - 6m - 3 < 0$$

$$3 - \sqrt{12} < m < 3 + \sqrt{12} \quad \dots \text{(iii)}$$

$\therefore$  Common interval is

$$3 - \sqrt{12} < m < 3 + \sqrt{12}$$

$\therefore$  Integral value of  $m$  {0,1,2,3,4,5,6}

28. In a game, a man wins Rs. 100 if he gets 5 or 6 on a throw of a fair die and loses Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is :

- (1)  $\frac{400}{3}$  gain            (2)  $\frac{400}{3}$  loss  
(3) 0                      (4)  $\frac{400}{9}$  loss

Ans. (3)

Expected Gain/ Loss =

$$= w \times 100 + Lw(-50 + 100) + L^2w(-50 - 50 + 100) + L^3(-150)$$

$$= \frac{1}{3} \times 100 + \frac{2}{3} \cdot \frac{1}{3} (50) + \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right) (0) +$$

$$\left(\frac{2}{3}\right)^3 (-150) = 0$$

here  $w$  denotes probability that outcome 5 or 6 (

$$w = \frac{2}{6} = \frac{1}{3})$$

here  $L$  denotes probability that outcome

$$1, 2, 3, 4 \quad (L = \frac{4}{6} = \frac{2}{3})$$

29. If a curve passes through the point  $(1, -2)$  and has slope of the tangent at any point  $(x, y)$  on it as

$$\frac{x^2 - 2y}{x}, \text{ then the curve also passes through the}$$

point :

- (1)  $(-\sqrt{2}, 1)$             (2)  $(\sqrt{3}, 0)$   
(3)  $(-1, 2)$             (4)  $(3, 0)$

Ans. (2)

$$\frac{dy}{dx} = \frac{x^2 - 2y}{x} \quad \text{(Given)}$$

$$\frac{dy}{dx} + 2\frac{y}{x} = x$$

$$\text{I.F} = e^{\int \frac{2}{x} dx} = x^2$$

$$\therefore y \cdot x^2 = \int x \cdot x^2 dx + C$$

$$= \frac{x^4}{y} + C$$

$$\text{hence it passes through } (1, -2) \Rightarrow C = -\frac{9}{4}$$

$$\therefore yx^2 = \frac{x^4}{4} - \frac{9}{4}$$

Now check option(s), Which is satisfied by option (ii)

30. Let  $Z_1$  and  $Z_2$  be two complex numbers satisfying  $|Z_1| = 9$  and  $|Z_2 - 3 - 4i| = 4$ . Then the minimum value of  $|Z_1 - Z_2|$  is :

- (1) 0            (2) 1            (3)  $\sqrt{2}$             (4) 2

Ans. (1)

$$|z_1| = 9, \quad |z_2 - (3 + 4i)| = 4$$

$$C_1 (0, 0) \text{ radius } r_1 = 9$$

$$C_2 (3, 4), \text{ radius } r_2 = 4$$

$$C_1 C_2 = |r_1 - r_2| = 5$$

$\therefore$  Circle touches internally

$$\therefore |z_1 - z_2|_{\min} = 0$$