TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

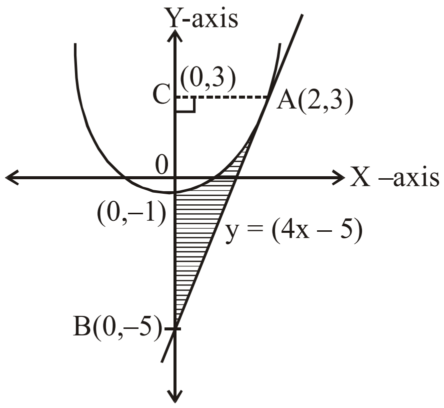
MATHEMATICS

1. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is :

(1) $\frac{14}{3}$ (2) $\frac{56}{3}$ (3) $\frac{8}{3}$ (4) $\frac{32}{3}$

Ans. (3)

Sol.



Equation of tangent at (2,3) on $y = x^2 - 1$, is $y = (4x - 5)$ ....(i)

$\therefore$ Required shaded area

$$= ar(\Delta ABC) - \int_{-1}^{3}\sqrt{y+1}\,dy$$

$$= \frac{1}{2}.(8).(2) - \frac{2}{3}\left((y+1)^{3/2}\right)_{-1}^{3}$$

$$= 8 - \frac{16}{3} = \frac{8}{3} \text{ (square units)}$$

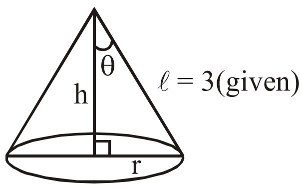
2. The maximum volume (in cu. m) of the right circular cone having slant height 3m is :

(1) $3\sqrt{3}\ \pi$ (2) $6\ \pi$

(3) $2\sqrt{3}\ \pi$ (4) $\frac{4}{3}\ \pi$

Ans. (3)

Sol.



$\therefore$ $h = 3\cos\theta$

$r = 3\sin\theta$

Now,

$$V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}(9\sin^2\theta).(3\cos\theta)$$

$$\therefore \frac{dV}{d\theta} = 0 \Rightarrow \sin\theta = \sqrt{\frac{2}{3}}$$

Also, $\left.\frac{d^2V}{d\theta^2}\right]_{\sin\theta=\sqrt{\frac{2}{3}}}$ = negative

$\Rightarrow$ Volume is maximum,

when $\sin\theta = \sqrt{\frac{2}{3}}$

$$\therefore V_{max}\left(\sin\theta = \sqrt{\frac{2}{3}}\right) = 2\sqrt{3}\pi \text{ (in cu. m)}$$

3. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral

$$\int x\sqrt{\frac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}}dx$$

is equal to :

(where c is a constant of integration)

(1) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$

(2) $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right| + c$

(3) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right| + c$

(4) $\frac{1}{2}\log_e\left|\sec(x^2-1)\right| + c$

Ans. (1)

Sol. Put $(x^2 - 1) = 1$

$\Rightarrow 2x\,dx = dt$

$$\therefore I = \frac{1}{2}\int\sqrt{\frac{1-\cos t}{1+\cos t}}\,dt$$

$$= \frac{1}{2}\int\tan\left(\frac{t}{2}\right)dt$$

$$= \ln\left|\sec\frac{t}{2}\right| + c$$

$$I = \ln\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$$

4. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to :

(1) 512 (2) –512 (3) –256 (4) 256

Ans. (3)

Sol. We have

$(x + 1)^2 + 1 = 0$

$\Rightarrow (x + 1)^2 - (i)^2 = 0$

$\Rightarrow (x + 1 + i)(x + 1 - i) = 0$

$\therefore\ x = \underbrace{-(1+i)}_{\alpha(\text{let})}\ \underbrace{-(1-i)}_{\beta(\text{let})}$

So, $\alpha^{15} + \beta^{15} = (\alpha^2)^7 \alpha + (\beta^2)^7 \beta$

$= -128(-i + 1 + i + 1)$

$= -256$

5. If $y = y(x)$ is the solution of the differential equation,

$x\dfrac{dy}{dx} + 2y = x^2$ satisfying

$y(1) = 1$, then $y\left(\dfrac{1}{2}\right)$ is equal to :

(1) $\dfrac{7}{64}$ (2) $\dfrac{13}{16}$ (3) $\dfrac{49}{16}$ (4) $\dfrac{1}{4}$

Ans. (3)

Sol. $\dfrac{dy}{dx} + \left(\dfrac{2}{x}\right)y = x$

$\Rightarrow$ I.F. $= x^2$

$\therefore\ yx^2 = \dfrac{x^4}{4} + \dfrac{3}{4}$ (As, y(1) = 1)

$\therefore\ y\left(x = \dfrac{1}{2}\right) = \dfrac{49}{16}$

6. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is:

(1) $2\sqrt{3}\,y = 12x + 1$

(2) $2\sqrt{3}\,y = -x - 12$

(3) $\sqrt{3}\,y = x + 3$

(4) $\sqrt{3}\,y = 3x + 1$

Ans. (3)

Sol. Let equation of tangent to the parabola $y^2 = 4x$ is

$y = mx + \dfrac{1}{m}$,

$\Rightarrow m^2x - ym + 1 = 0$ is tangent to $x^2 + y^2 - 6x = 0$

$\Rightarrow \dfrac{|3m^2 + 1|}{\sqrt{m^4 + m^2}} = 3$

$m = \pm\dfrac{1}{\sqrt{3}}$

$\Rightarrow$ tangent are $x + \sqrt{3}y + 3 = 0$

and $x - \sqrt{3}y + 3 = 0$

7. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is:

(1) 200 (2) 300 (3) 500 (4) 350

Ans. (2)

Sol. Required number of ways

= Total number of ways – When A and B are always included.

$= {}^5C_2 \cdot {}^7C_3 - {}^5C_1\,{}^5C_2 = 300$

8. Three circles of radii a, b, c(a < b < c) touch each other externally. If they have x-axis as a common tangent, then :

(1) $\dfrac{1}{\sqrt{a}} = \dfrac{1}{\sqrt{b}} + \dfrac{1}{\sqrt{c}}$

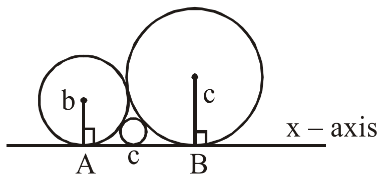
(2) a, b, c are in A. P.

(3) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A. P.

(4) $\dfrac{1}{\sqrt{b}} = \dfrac{1}{\sqrt{a}} + \dfrac{1}{\sqrt{c}}$

Ans. (1)

Sol.



AB = AC + CB

$\sqrt{(b+c)^2 - (b-c)^2}$

$= \sqrt{(b+a)^2 - (b-a)^2} + \sqrt{(a+c)^2 - (a-c)^2}$

$\sqrt{bc} = \sqrt{ab} + \sqrt{ac}$

$\dfrac{1}{\sqrt{a}} = \dfrac{1}{\sqrt{c}} + \dfrac{1}{\sqrt{b}}$

9. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to :

(1) 14 (2) 6

(3) 4 (4) 8

Ans. (4)

Sol. $\frac{2^{403}}{15} = \frac{2^3.(2^4)^{100}}{15} = \frac{8}{15}(15+1)^{100}$

$= \frac{8}{15}(15\lambda+1) = 8\lambda + \frac{8}{15}$

$\because$ 8λ is integer

$\Rightarrow$ fractional part of $\frac{2^{403}}{15}$ is $\frac{8}{15} \Rightarrow k = 8$

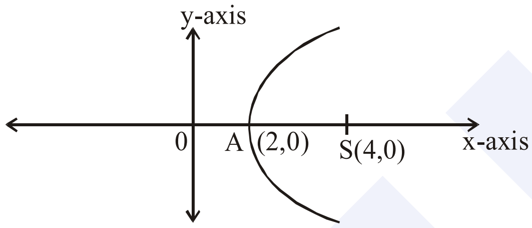
10. Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it ?

(1) (4, –4) (2) $(5, 2\sqrt{6})$

(3) (8, 6) (4) $6, 4\sqrt{2}$

Ans. (3)

Sol.



equation of parabola is

$y^2 = 8(x - 2)$

(8, 6) does not lie on parabola.

11. The plane through the intersection of the planes x + y + z = 1 and 2x + 3y – z + 4 = 0 and parallel to y-axis also passes through the point :

(1) (–3, 0, –1) (2) (3, 3, –1)

(3) (3, 2, 1) (4) (–3, 1, 1)

Ans. (3)

Sol. Equation of plane

$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z - 1 + 4\lambda = 0$

dr's of normal of the plane are

$1 + 2\lambda,\ 1 + 3\lambda,\ 1 - \lambda$

Since plane is parallel to y - axis, $1 + 3\lambda = 0$

$\Rightarrow \lambda = -1/3$

So the equation of plane is

$x + 4z - 7 = 0$

Point (3, 2, 1) satisfies this equation

Hence Answer is (3)

12. If a, b and c be three distinct real numbers in G. P. and a + b + c = xb, then x cannot be :

(1) 4 (2) –3 (3) –2 (4) 2

Ans. (4)

Sol. $\frac{b}{r}, b, br \rightarrow$ G.P. $(|r| \neq 1)$

given a + b + c = xb

$\Rightarrow b/r + b + br = xb$

$\Rightarrow b = 0$ (not possible)

or $1 + r + \frac{1}{r} = x \Rightarrow x - 1 = r + \frac{1}{r}$

$\Rightarrow x - 1 > 2$ or $x - 1 < -2$

$\Rightarrow x > 3$ or $x < -1$

So x can't be '2'

13. Consider the set of all lines px + qy + r = 0 such that 3p + 2q + 4r = 0. Which one of the following statements is true ?

(1) The lines are all parallel.

(2) Each line passes through the origin.

(3) The lines are not concurrent

The lines are concurrent at the point

(4) $\left(\frac{3}{4}, \frac{1}{2}\right)$

Ans. (4)

Sol. Given set of lines px + qy + r = 0

given condition 3p + 2q + 4r = 0

$\Rightarrow \frac{3}{4}p + \frac{1}{2}q + r = 0$

$\Rightarrow$ All lines pass through a fixed point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

14. The system of linear equations.

x + y + z = 2

2x + 3y + 2z = 5

$2x + 3y + (a^2 - 1)z = a + 1$

(1) has infinitely many solutions for a = 4

(2) is inconsistent when $|a| = \sqrt{3}$

(3) is inconsistent when a = 4

(4) has a unique solution for $|a| = \sqrt{3}$

Ans. (2)

Sol. $D=\begin{vmatrix}1&1&1\\2&3&2\\2&3&a^2-1\end{vmatrix}=a^2-3$

$D_1=\begin{vmatrix}2&1&1\\5&3&2\\a+1&3&a^2-1\end{vmatrix}=a^2-a+1$

$D_2=\begin{vmatrix}1&2&1\\2&5&2\\2&a+1&a^2-1\end{vmatrix}=a^2-3$

$D_3=\begin{vmatrix}1&1&2\\2&3&5\\2&3&a+1\end{vmatrix}=a-4$

$D = 0$ at $|a| = \sqrt{3}$ but $D_3 = \pm\sqrt{3}-4\neq 0$

So the system is Inconsistant for $|a|=\sqrt{3}$

15. Let $\vec{a}=\hat{i}-\hat{j}, \vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}+\vec{b}=\vec{0}$ and $\vec{a}.\vec{c}=4$, then $|\vec{c}|^2$ is equal to :-

(1) $\frac{19}{2}$ (2) 8 (3) $\frac{17}{2}$ (4) 9

Ans. (1)

Sol. $\vec{a}\times\vec{c}=-\vec{b}$

$(\vec{a}\times\vec{c})\times\vec{a}=-\vec{b}\times\vec{a}$

$\Rightarrow (\vec{a}\times\vec{c})\times\vec{a}=\vec{a}\times\vec{b}$

$\Rightarrow (\vec{a}.\vec{a})\vec{c}-(\vec{c}.\vec{a})\vec{a}=\vec{a}\times\vec{b}$

$\Rightarrow 2\vec{c}-4\vec{a}=\vec{a}\times\vec{b}$

Now $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&0\\1&1&1\end{vmatrix}=-\hat{i}-\hat{j}+2\hat{k}$

So, $2\vec{c}=4\hat{i}-4\hat{j}-\hat{i}-\hat{j}+2\hat{k}$

$=3\hat{i}-5\hat{j}+2\hat{k}$

$\Rightarrow \vec{c}=\frac{3}{2}\hat{i}-\frac{5}{2}\hat{j}+\hat{k}$

$|\vec{c}|=\sqrt{\frac{9}{4}+\frac{25}{4}+1}=\sqrt{\frac{38}{4}}=\sqrt{\frac{19}{2}}$

$|\vec{c}|^2=\frac{19}{2}$

16. Let $a_1, a_2,\ldots,a_{30}$ be an A. P., $S=\sum_{i=1}^{30}a_i$ and $T=\sum_{i=1}^{15}a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to :

(1) 57 (2) 47 (3) 42 (4) 52

Ans. (4)

Sol. $S = a_1 + a_2 + \ldots\ldots + a_{30}$

$S=\frac{30}{2}[a_1+a_{30}]$

$S = 15(a_1 + a_{30}) = 15 (a_1 + a_1 + 29d)$

$T = a_1 + a_3 + \ldots.. + a_{29}$

$= (a_1) + (a_1 + 2d) \ldots.. + (a_1 + 28d)$

$= 15a_1 + 2d(1 + 2 + \ldots.. + 14)$

$T = 15a_1 + 210\ d$

Now use $S - 2T = 75$

$\Rightarrow 15 (2a_1 + 29d) - 2 (15a_1 + 210\ d) = 75$

$\Rightarrow d = 5$

Given $a_5 = 27 = a_1 + 4d \Rightarrow a_1 = 7$

Now $a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$

17. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is:

(1) 22 (2) 20 (3) 16 (4) 18

Ans. (2)

Sol. Given $\bar{x}=\frac{\Sigma x_i}{5}=150$

$\Rightarrow \sum_{i=1}^{5}x_i=750$ .....(i)

$\frac{\Sigma x_i^2}{5}-(\bar{x})^2=18$

$\frac{\Sigma x_i^2}{5}-(150)^2=18$

$\Sigma x_i^2=112590$ ....(ii)

Given height of new student

$x_6 = 156$

Now, $\bar{x}_{new}=\frac{\sum_{i=1}^{6}x_i}{6}=\frac{750+156}{6}=151$

Also, New variance $=\frac{\sum_{i=1}^{6}x_i^2}{6}-(\bar{x}_{new})^2$

$=\frac{112590+(156)^2}{6}-(151)^2$

$=22821-22801 = 20$

18. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P(X = 2) equals :

(1) 52/169 (2) 25/169

(3) 49/169 (4) 24/169

Ans. (2)

Sol. Two cards are drawn successively with replacement

4 Aces 48 Non Aces

$$P(x=1)=\frac{{}^{4}C_1}{{}^{52}C_1}\times\frac{48C_1}{52C_1}+\frac{48C_1}{52C_1}\times\frac{4C_1}{52C_1}=\frac{24}{169}$$

$$P(x=2)=\frac{{}^{4}C_1}{{}^{52}C_1}\times\frac{{}^{4}C_1}{{}^{52}C_1}=\frac{1}{169}$$

$$P(x = 1) + P(x = 2) = \frac{25}{169}$$

19. For $x \in R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ then J(x) is equal to :-

(1) $f_3(x)$ (2) $f_1(x)$

(3) $f_2(x)$ (4) $\frac{1}{x}f_3(x)$

Ans. (1)

Sol. Given $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x)=\frac{1}{1-x}$

$$(f_2\circ J\circ f_1)(x)=f_3(x)$$

$$f_2\circ\left(J\left(f_1(x)\right)\right) = f_3(x)$$

$$f_2\circ\left(J\left(\frac{1}{x}\right)\right)=\frac{1}{1-x}$$

$$1-J\left(\frac{1}{x}\right)=\frac{1}{1-x}$$

$$J\left(\frac{1}{x}\right)=1-\frac{1}{1-x}=\frac{-x}{1-x}=\frac{x}{x-1}$$

Now $x\rightarrow\frac{1}{x}$

$$J(x)=\frac{\frac{1}{x}}{\frac{1}{x}-1}=\frac{1}{1-x}=f_3(x)$$

20. Let

$$A=\left\{\theta\in\left(-\frac{\pi}{2},\pi\right):\frac{3+2i\sin\theta}{1-2i\sin\theta}\text{ is purely imaginary}\right\}$$

Then the sum of the elements in A is :

(1) $\frac{5\pi}{6}$ (2) $\frac{2\pi}{3}$

(3) $\frac{3\pi}{4}$ (4) π

Ans. (2)

Sol. Given $z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely img

so real part becomes zero.

$$z=\left(\frac{3+2i\sin\theta}{1-2i\sin\theta}\right)\times\left(\frac{1+2i\sin\theta}{1+2i\sin\theta}\right)$$

$$z=\frac{(3-4\sin^2\theta)+i(8\sin\theta)}{i+4\sin^2\theta}$$

Now Re(z) = 0

$$\frac{3-4\sin^2\theta}{1+4\sin^2\theta}=0$$

$$\sin^2\theta = \frac{3}{4}$$

$$\sin\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow \theta = --\frac{\pi}{3},\frac{\pi}{3},\frac{2\pi}{3}$$

$$\because \theta\in\left(-\frac{\pi}{2},\pi\right)$$

then sum of the elements in A is

$$-\frac{\pi}{3}+\frac{\pi}{3}+\frac{2\pi}{3}=\frac{2\pi}{3}$$

21. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then |tan θ| is equal to :

(1) 4/9 (2) 7/17

(3) 8/17 (4) 8/15

Ans. (4)

Sol. Point of intersection is P(2,6).

Also, $m_1 = \left(\frac{dy}{dx}\right)_{P(2,6)} = -2x = -4$

$m_2 = \left(\frac{dy}{dx}\right)_{P(2,6)} = 2x = 4$

$$\therefore |\tan\theta| = \left|\frac{m_1-m_2}{1+m_1m_2}\right| = \frac{8}{15}$$

22. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to :

(1) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

(3) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Ans. **(1)**

Sol. Here, $AA^T = I$

$\Rightarrow A^{-1} = A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Also, $A^{-n} = \begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix}$

$\therefore A^{-50} = \begin{bmatrix} \cos(50)\theta & \sin(50)\theta \\ -\sin(50)\theta & \cos(50)\theta \end{bmatrix}$

$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

23. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval :

(1) (2, 3] (2) (3, ∞)

(3) (3/2, 2] (4) (1, 3/2]

Ans. **(2)**

Sol. $e = \sqrt{1 + \tan^2\theta} = \sec\theta$

As, $\sec\theta > 2 \Rightarrow \cos\theta < \frac{1}{2}$

$\Rightarrow \theta \in (60^\circ, 90^\circ)$

Now, $\ell(L{\cdot}R) = \frac{2b^2}{a} = 2\frac{(1-\cos^2\theta)}{\cos\theta}$

$= 2(\sec\theta - \cos\theta)$

Which is strictly increasing, so

ℓ (L.R) $\in (3, \infty)$.

24. The equation of the line passing through (–4, 3, 1), parallel to the plane x + 2y – z – 5 = 0 and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:

(1) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

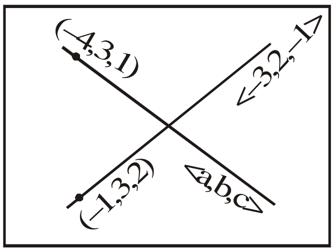
(2) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(3) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

(4) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Ans. **(2)**

Sol.



Normal vector of plane containing two intersecting lines is parallel to vector.

$(\vec{V}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ -3 & 2 & -1 \end{vmatrix}$

$= -2\hat{i} + 6\hat{k}$

$\therefore$ Required line is parallel to vector

$(\vec{V}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 0 & 6 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$

$\Rightarrow$ Required equation of line is

$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

25. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals :

(1) $13 - 4\cos^6\theta$
(2) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$
(3) $13 - 4\cos^2\theta + 6\cos^4\theta$
(4) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

Ans. **(1)**

Sol. We have,

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$
$= 3(1 - \sin2\theta)^2 + 6(1 + \sin2\theta) + 4\sin^6\theta$
$= 3(1 - 2\sin2\theta + \sin^2 2\theta) + 6 + 6\sin2\theta + 4\sin^6\theta$
$= 9 + 12\sin^2\theta \cdot \cos^2\theta + 4(1 - \cos^2\theta)^3$
$= 13 - 4\cos^6\theta$

26. If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$ then x is equal to :

(1) $\frac{\sqrt{145}}{12}$ (2) $\frac{\sqrt{145}}{10}$

(3) $\frac{\sqrt{146}}{12}$ (4) $\frac{\sqrt{145}}{11}$

Ans. **(1)**

Sol. $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2} \quad \left(x > \frac{3}{4}\right)$

$\cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{2}{3x}\right)$

$\cos^{-1}\left(\frac{3}{4x}\right) = \sin^{-1}\left(\frac{2}{3x}\right)$

$\cos\left(\cos^{-1}\left(\frac{3}{4x}\right)\right) = \cos\left(\sin^{-1}\frac{2}{3x}\right)$

$\frac{3}{4x} = \frac{\sqrt{9x^2 - 4}}{3x}$

$\frac{81}{16} + 4 = 9x^2$

$x^2 = \frac{145}{16 \times 9} \Rightarrow x = \frac{\sqrt{145}}{12}$

27. The value of $\int_0^{\pi} |\cos x|^3\, dx$

(1) 2/3 (2) 0
(3) –4/3 (4) 4/3

Ans. **(4)**

Sol. $\int_0^{\pi} |\cos x|^3\, dx = \int_0^{\pi/2} \cos^3 x\, dx - \int_{\pi/2}^{\pi} \cos^3 x\, dx$

$= \int_0^{\pi/2} \left(\frac{\cos3x + 3\cos x}{4}\right)dx - \int_{\pi/2}^{\pi} \left(\frac{\cos3x + 3\cos x}{4}\right)dx$

$= \frac{1}{4}\left[\left(\frac{\sin3x}{3} + 3\sin x\right)_0^{\pi/2} - \left(\frac{\sin3x}{3} + 3\sin x\right)_{\pi/2}^{\pi}\right]$

$= \frac{1}{4}\left[\left(\frac{-1}{3} + 3\right) - (0+0) - \left\{(0+0) - \left(\frac{-1}{3} + 3\right)\right\}\right]$

$= \frac{4}{3}$

28. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is:

(1) $(\wedge, \vee)$

(2) $(\vee, \vee)$

(3) $(\wedge, \wedge)$

(4) $(\vee, \wedge)$

Ans. **(1)**

Sol. (p ⊕ q) ∧ (~p □ q) ≡ p ∧ q (given)

p	q	~ p	p ∧ q	p ∨ q	~ p ∨ q	~ p ∧ q	(p ∧ q) ∧ (~ p ∨ q)
T	T	F	T	T	T	F	T
T	F	F	F	T	F	F	F
F	T	T	F	T	T	T	F
F	F	T	F	F	T	F	F

from truth table (⊕, □) = (∧ , ∨)

29. $\lim_{y\to 0}\dfrac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

(1) exists and equals $\dfrac{1}{4\sqrt{2}}$

(2) does not exist

(3) exists and equals $\dfrac{1}{2\sqrt{2}}$

(4) exists and equals $\dfrac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

Ans. (1)

Sol. $\lim_{y\to 0}\dfrac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

$$= \lim_{y\to 0}\frac{1+\sqrt{1+y^4}-2}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$= \lim_{y\to 0}\frac{\left(\sqrt{1+y^4}-1\right)\left(\sqrt{1+y^4}+1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$= \lim_{y\to 0}\frac{1+y^4-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$= \lim_{y\to 0}\frac{1}{\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)} = \frac{1}{4\sqrt{2}}$$

30. Let f : R → R be a function defined as :

$$f(x) = \begin{cases} 5, & \text{if } x \le 1 \\ a+bx, & \text{if } 1 < x < 3 \\ b+5x, & \text{if } 3 \le x < 5 \\ 30, & \text{if } x \ge 5 \end{cases}$$

Then, f is :

(1) continuous if a = 5 and b = 5

(2) continuous if a = –5 and b = 10

(3) continuous if a = 0 and b = 5

(4) not continuous for any values of a and b

Ans. (4)

Sol. $f(x) = \begin{cases} 5 & \text{if } x \le 1 \\ a+bx & \text{if } 1 < x < 3 \\ b+5x & \text{if } 3 \le x < 5 \\ 30 & \text{if } x \ge 5 \end{cases}$

$f(1) = 5$, $f(1^-) = 5$, $f(1^+) = a + b$

$f(3^-) = a + 3b$, $f(3) = b + 15$, $f(3^+) = b + 15$

$f(5^-) = b + 25$; $f(5) = 30$ $f(5^+) = 30$

from above we concluded that f is not continuous for any values of a and b.